AN INVESTIGATION OF THE VIRTUAL MASS OF A CYLINDER VIBRATING IN WATER

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1953

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of a

Cylinder Vibrating in Water

by

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Submitted to the Department of Naval Architecture and Marine Engineering on May 25, 1953 in partial fulfillment of the requirements for the degree of Naval Engineer.

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Professor F. M. Lewis,—Thesis Supervisor

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Chairman of Department Committee on Graduate Students

ABSTRACT

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DAVID A. ROGERS, LIEUTENANT, U.S. NAVY and MACLEAN C. SHAKSHOBER, LIEUTENANT, U.S. NAVY

Submitted to the Department of Naval Architecture and Marine Engineering on May 25, 1953 in partial fulfillment of the requirements for the degree of Naval Engineer.

The object of this thesis is to experimentally investigate the virtual mass of a hollow cylinder vibrating in water.

A lucite cylinder was magnetically vibrated in air and water at various length to diameter ratios and the frequency of vibration for as many modes as possible, up to five, recorded. No attempt was made to measure amplitude. The ratio of added water mass to displaced water mass was computed from the frequencies and compared with analytical results.

The investigation shows that end effects have a very great influence on virtual mass. As the length to diameter ratio is decreased, the added virtual water mass is decreased. There is also a decrease in virtual water mass as frequency is increased at constant length to diameter ratios.

A ratio of the measured virtual water mass to analytical, called K, was computed and found to be a function of L/D and mode number.

It is recommended that further investigations using bodies of revolution whose ends have zero area such as ellipsoids be made. It would be desirable to use equipment to permit measuring amplitude as well as frequency.

Where data is available, it is recommended that an attempt to calculate the frequencies of an actual hull such as a submarine be made, correcting the analytical virtual water mass by the applicable K values.

Thesis Supervisor: Professor F. M. Lewis
Title: Professor of Marine Engineering

Cambridge, Massachusetts
May 25, 1953

Professor Earl B. Millard
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Naval Engineer, we herewith submit a thesis entitled "An Investigation of the Virtual Mass of a Cylinder Vibrating in Water."

Respectfully,

David A. Rogers
Lieutenant, U. S. Navy

MacLean C. Shakshober
Lieutenant, U. S. Navy

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[illegible]

I. INTRODUCTION

When immersed in a dense fluid, such as water, a body vibrates as though it had undergone an increase in mass. This increase in mass is due to the flow of fluid about the body as it moves.

Several analytical methods of computing the virtual mass of a body vibrating in a dense fluid have been presented.[1, 2] These methods are based on the assumption of potential flow about the body. Unless the body is of uniform shape with pointed ends, i.e. an ellipsoid, it is not presently possible to compute analytically the virtual mass because of the flow about the ends being highly rotational.

A submerged submarine must vibrate as a free-free body at a frequency determined by its virtual mass as described above. When vibrating horizontally, its virtual mass will be somewhat lower than that computed on the basis of no end losses because of flow about the ends.

Professor Frank M. Lewis has presented a method for determining the virtual mass based on an ellipsoid which may be corrected for other shapes.[1] Dr. H. M. Schauer of the Underwater Explosion Research Division, Norfolk Naval Shipyard, has done likewise for a cylinder with no end flow. Mr. E. B. Moullin and Mr. A. D. Browne, in a paper presented before the Cambridge Philosophical Society in 1928[3] gave the results of their investigation of the periods of a free-free bar of rectangular cross section vibrating in water. In their experiments they used long bars which had length to depth ratios of from 26 to 39. Using such long bars they found that flow about the ends did not have any appreciable effect on the virtual mass as analytically computed. However, they did not investigate lower length to depth ratios. They found that the virtual mass is not affected by depth when below about two diameters.

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When immersed in a dense fluid, such as water, a body vibrates as though it had undergone an increase in mass. This increase in mass is due to the flow of fluid about the body as it moves.

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The following is a report on the experimental determination of the virtual mass of a circular cylinder for several length to diameter ratios while vibrating in water.

II. PROCEDURE

In this section the steps followed to obtain the desired results are described. The procedure consists of two parts, experimental and analytical. Details of the procedure are presented in the Appendix.

Experimental Procedure

From "The Theory of Sound" by Rayleigh[4] the appropriate equations were used to obtain the nodal and anti-nodal points of a free-free bar. Using the frequency equation for a free-free bar

$$f = \frac{m_n^2}{2\pi}\left[\frac{EK^2g}{\gamma}\right]^{1/2} \tag{1}$$

the first five modes were computed to give an approximation of the natural frequencies.

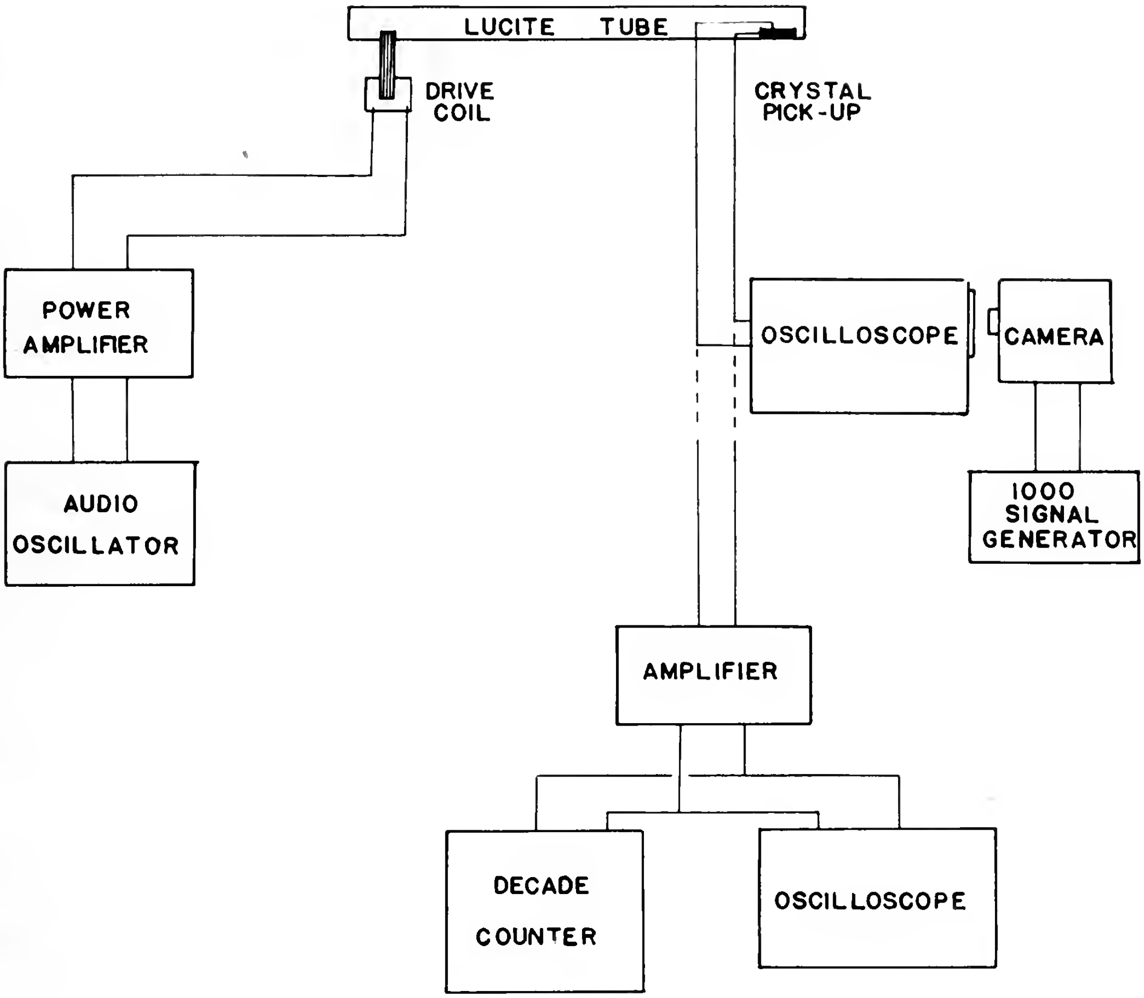
To vibrate the cylinder mechanically would require a motor with a speed range of 1800 to 60,000 revolutions per minute. For this reason, magnetic vibration of the bar was by far the preferred method. Schematics of the apparatus are shown in Figure I.

A Lucite plastic tube 52.7 inches long with an outer diameter of 2 inches and an inner diameter of 1.75 inches was used for the first test. On the end of the cylinder was wrapped some small diameter soft iron wire to permit magnetic excitation. The amount of wire was not great enough to affect the frequency or mass of the bar. By experimenting with various types of pickups, it was found that a seismic crystal gave the best results. The pickup was very light in weight and very sensitive to vibration. This particular pickup was a Brush seismic crystal used on the sounding board of an electric guitar. The pickup was mounted on the inside of the cylinder at the opposite end of

FIG. I

SCHEMATIC

MAY 8, 1953; MCS



the exciting wire. A small hole was drilled at a nodal point through which the wires from the pickup were run.

An audio oscillator with a frequency range of 20 to 20,000 cycles per second was used to drive an electro-magnet. The output of the audio oscillator was amplified in a power amplifier and this was used to drive the electro-magnet. The output of the crystal pickup was put into a cathode ray oscilloscope where the signal was peaked for resonance. Unfortunately, due to radiation, frequencies above 1600 cycles per second could not be detected.

Two different types of frequency measurement were employed to compute the frequency of vibration, both of which gave very accurate and like results. The first method was to take the output of the pickup and put it on the vertical plates of the CRO. When a resonant signal was obtained, a picture of the frequency was taken with no horizontal sweep. The camera used was a very high speed model with no shutter. The camera had a built-in timing light of 1000 cycles per second which showed on the film. The frequency was then computed from the developed film by counting the cycles.

The other method of determining the frequency was to take the amplified output of the crystal pickup and put it into an electronic decade counter.

To compute the air frequencies, the cylinder was suspended by strings located at the nodal points. The electro-magnet was placed as close as possible to the soft iron windings. To prevent banging of the cylinder, a rubber band was used as a standoff. The frequencies were recorded as described above.

For the water tests, the cylinder was immersed seven diameters in the towing tank in the M. I. T. Hydrodynamics Laboratory. This depth ensured that no surface effects would be present. The cylinder was anchored by two strings at the nodes.

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The same procedure was followed using cylinders of 40, 38, 30, 28, and 22 inches, and for a 42-inch bar with six-inch conical ends.

Analytical Procedure

The ratio of the added water mass to displaced water mass, M_v/M_e, was computed directly from the observed frequencies as explained in Details of Procedure.

A correction factor K was then calculated, where K is the ratio of measured M_v/M_e to that computed by Dr. H. M. Schauer.[2]

The same procedure was followed using cylinders of 40, 35, 30, 25, and 22 inches, and for a 42-inch bar with six-inch conical ends.

Analytical Procedure

The ratio of the added mass to the displaced water mass, M_a/M_w, was computed directly from the observed frequencies as explained in Details of Procedure.

A correction factor K was then calculated, where K is the ratio of measured M_a/M_w to that computed by Dr. [illegible] Lehmann.(2)

III. RESULTS

The results are plotted as shown in Figs. II through V.

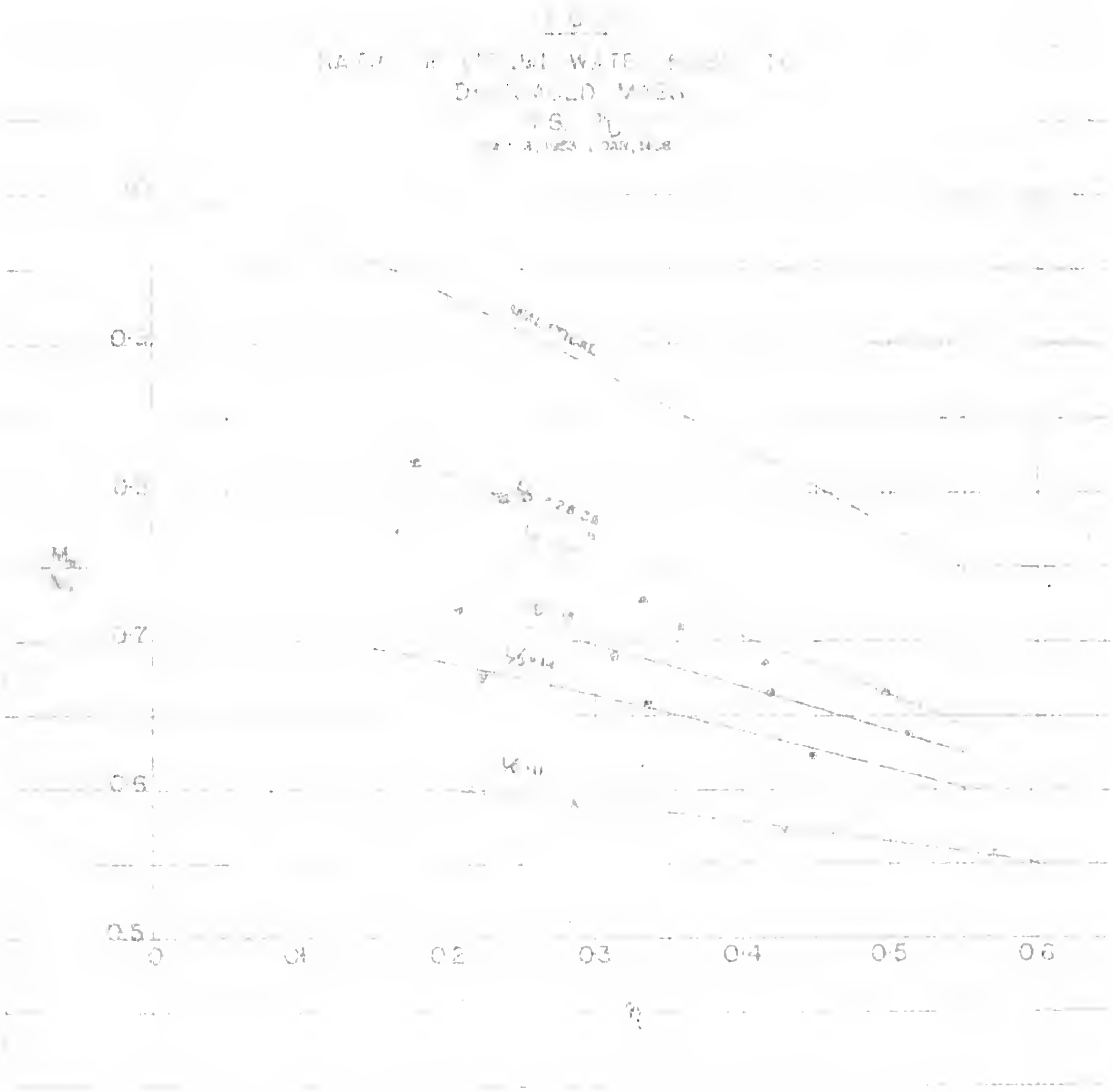
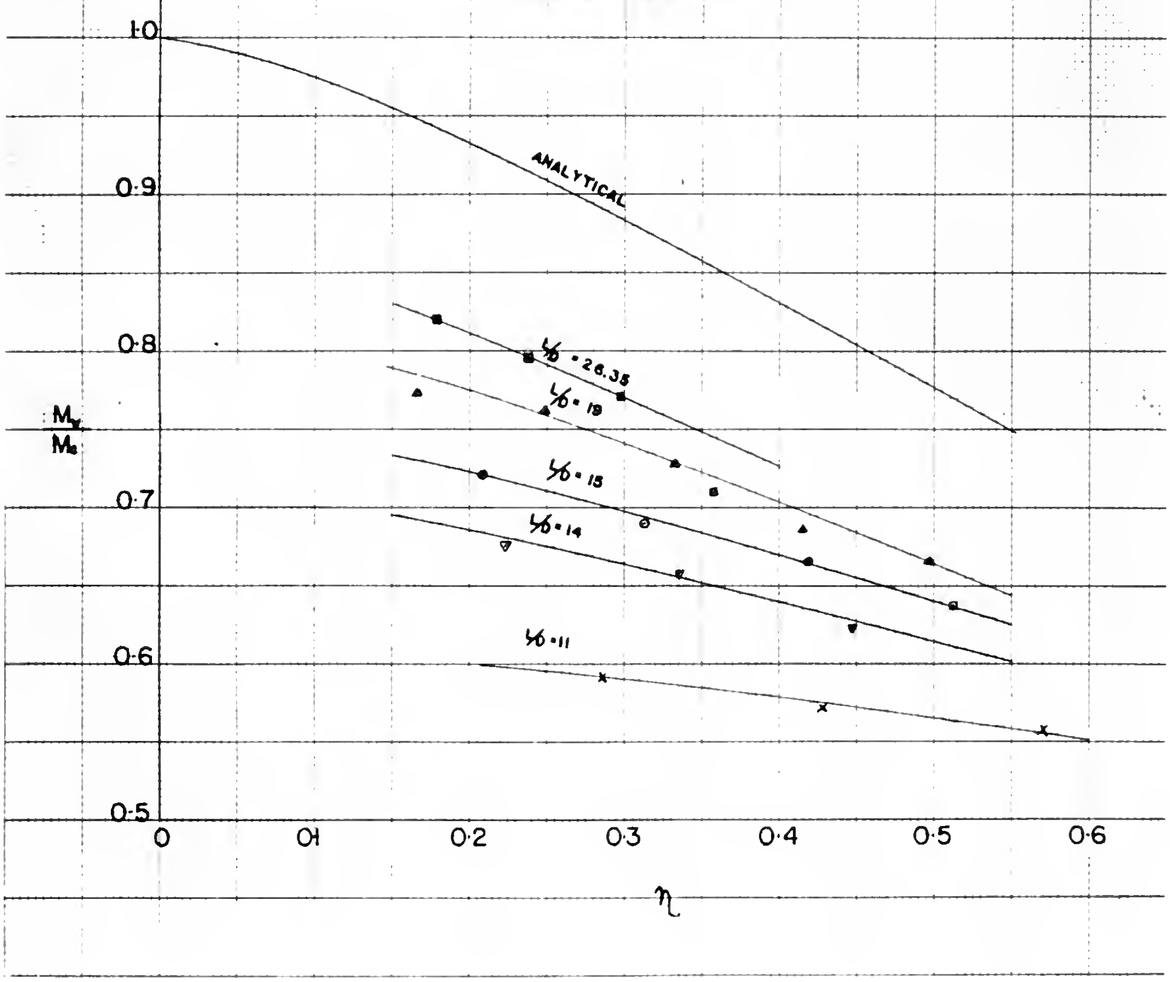


FIG. II

RATIO OF VIRTUAL WATER MASS TO DISPLACED MASS

V.S. η

MAY 8, 1953, DAR, MCS



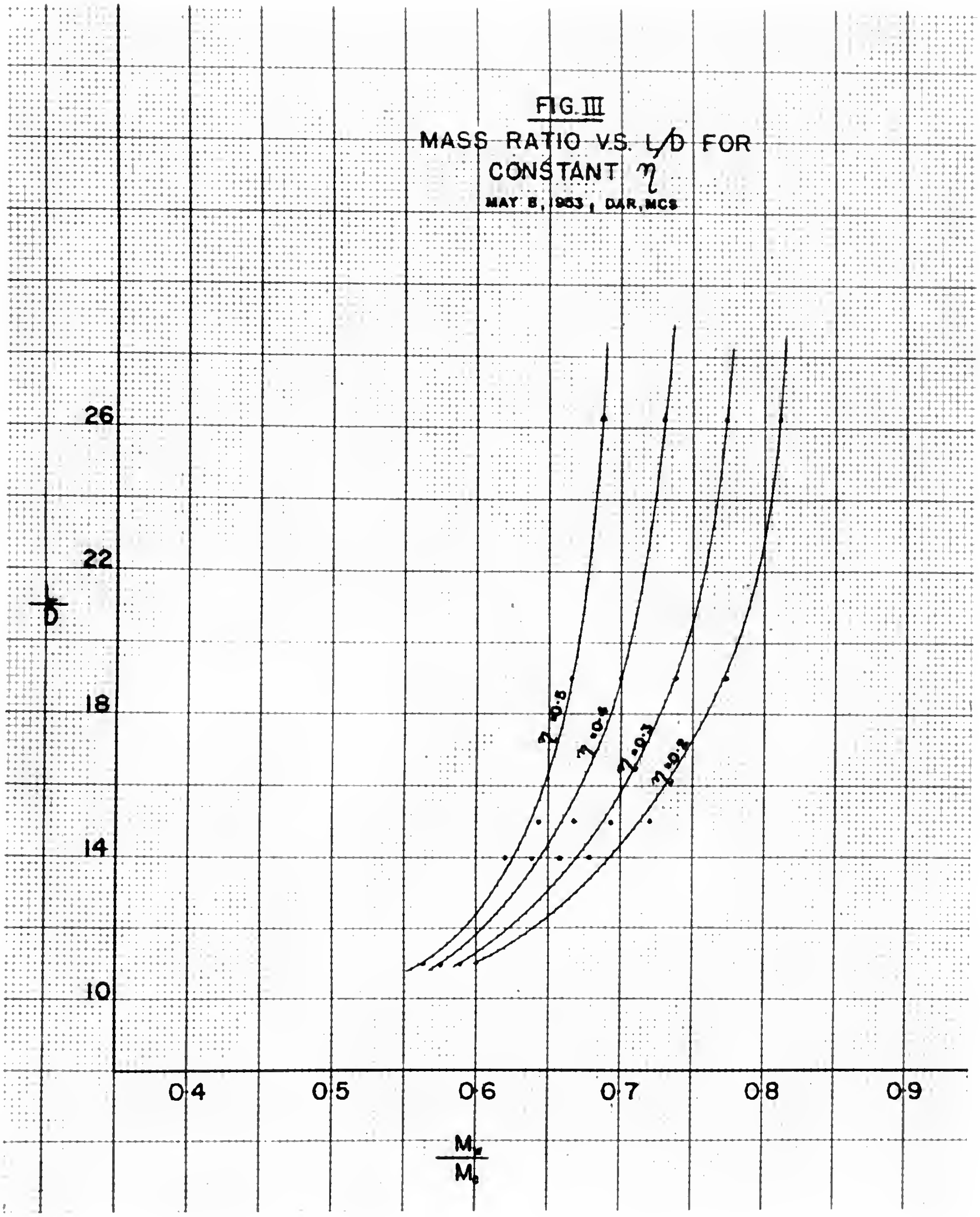


FIG. III
MASS RATIO VS. L/D FOR CONSTANT η
MAY 8, 1953, DAR, MCS
26
22
18
14
10
L/D
0·4
0·5
0·6
0·7
0·8
0·9
M/M
η=0.5
η=0.4
η=0.3
η=0.2

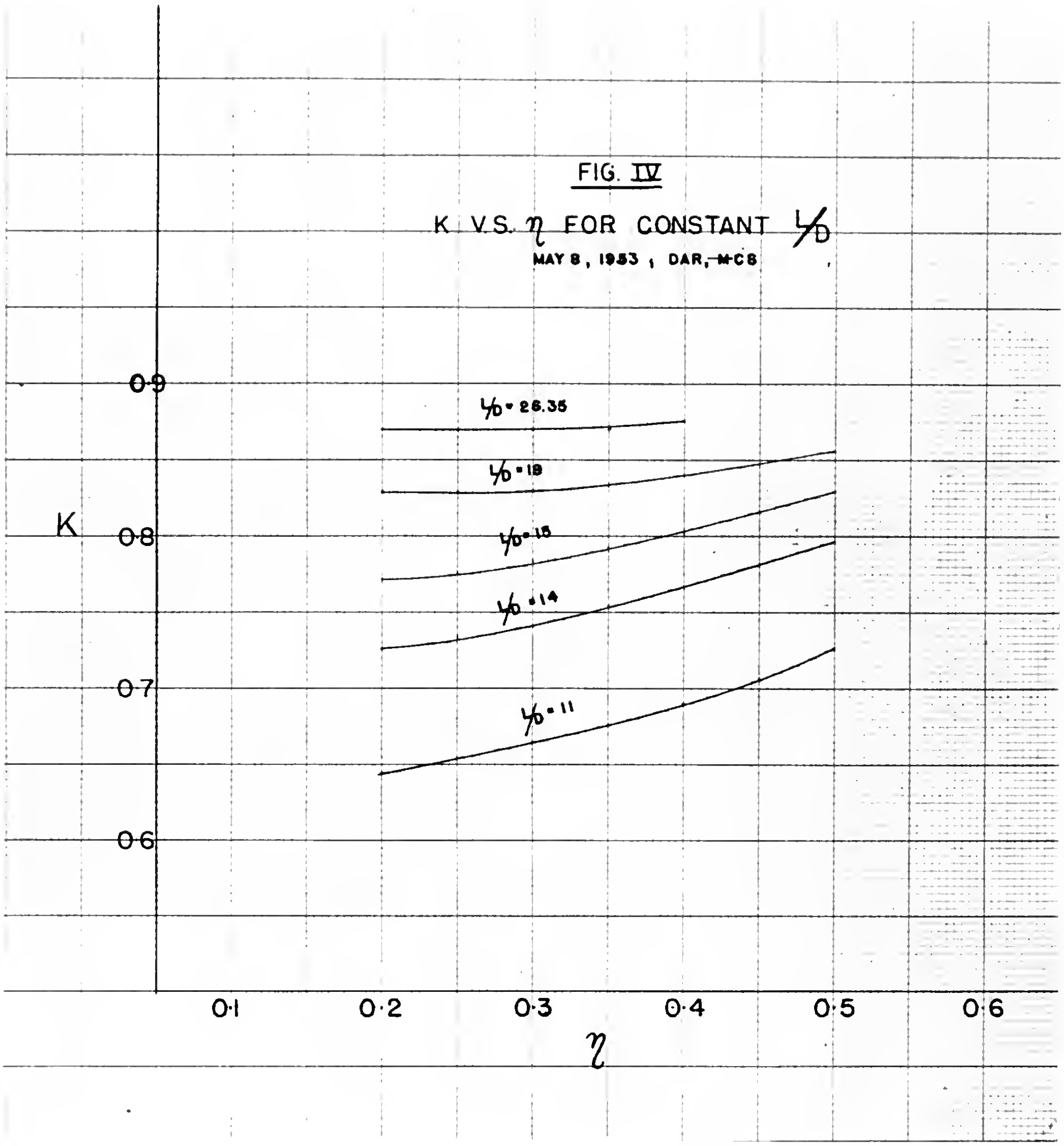


FIG. IV
K V.S. η FOR CONSTANT L/D
MAY 8, 1953, DAR, MCS
L/D = 26.35
L/D = 19
L/D = 15
L/D = 14
L/D = 11
K
0.9
0.8
0.7
0.6
0.1
0.2
0.3
0.4
0.5
0.6
η

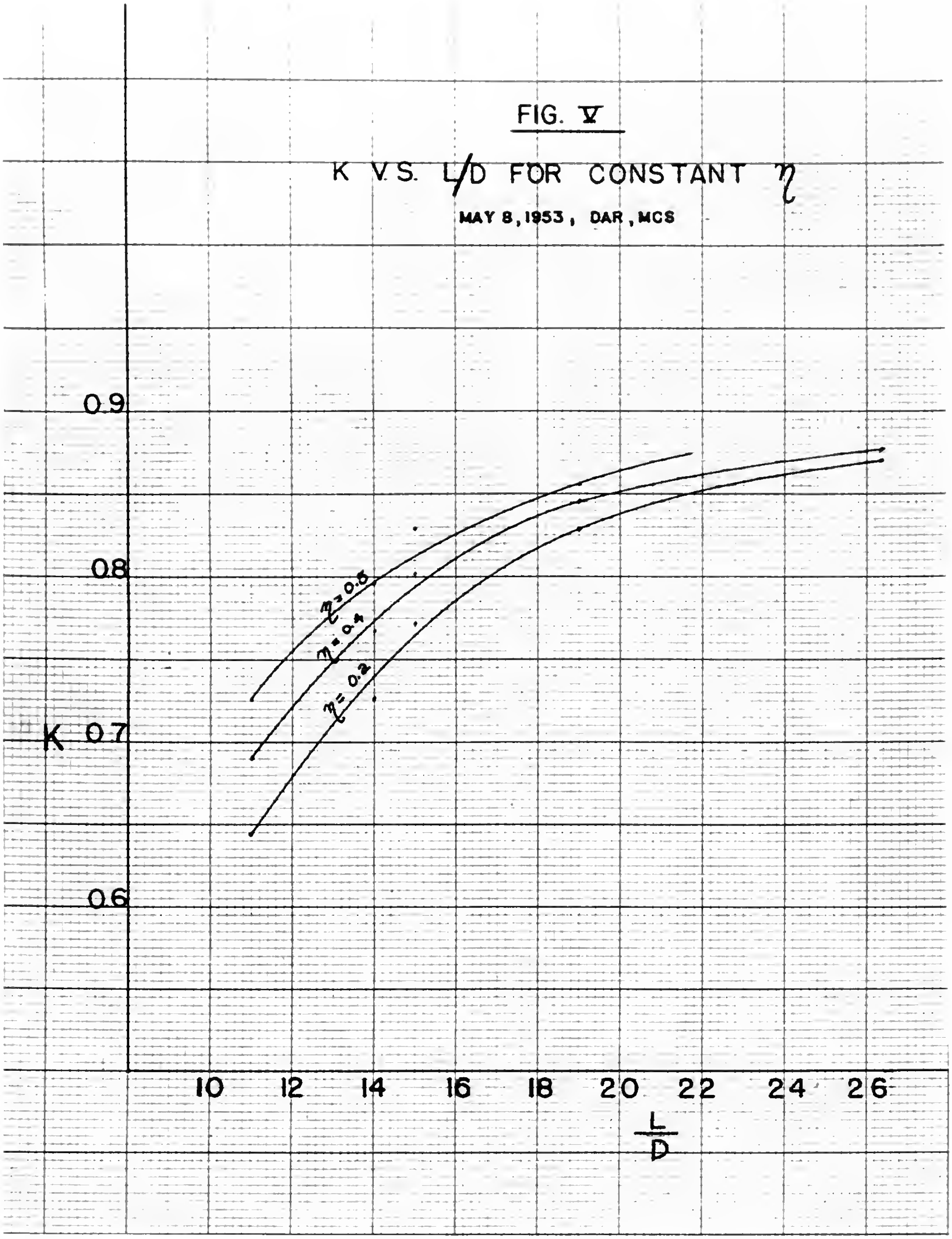


FIG. V
K V.S. L/D FOR CONSTANT η
MAY 8, 1953, DAR, MCS
η = 0.5
η = 0.4
η = 0.2
K
0.9
0.8
0.7
0.6
10
12
14
16
18
20
22
24
26
L/D

IV. DISCUSSION OF RESULTS

The results shown plotted in figures II through V indicate that the ends have a large influence on the value of the ratio of added water mass to the displaced water mass, i.e. M_w/M_c. As the length to diameter ratio is decreased, the value of K, (the ratio of measured M_w/M_c to the M_w/M_c calculated by Dr. Schauer), decreases rapidly for a given value of η. However, K increases for increasing η at constant L/D.

K is based on an analytical curve which assumes a potential flow, and therefore irrotational, with no flow about the ends. Thus K may be considered a measure of the end flow and of rotationality. For a cylinder with flat ends a large portion of K can probably be attributed to flow of the fluid about the ends.

Just how much of the difference from analytical values can be attributed to end flow is difficult to say from these experiments. An inspection of the test results of the cylinder with conical ends indicates that end flow is not the only contributing factor since the area at the ends of the cones is substantially zero. However, due to the abrupt change in shape of the body where the cones are joined to the cylinder, a certain amount of spilling of fluid toward the ends will occur.

Due to the large effects of L/D on the value of M_w/M_c it appears that a direct application of Dr. Schauer's equation is not feasible unless the body is very long relative to its breadth.

Professor F. M. Lewis[1] has shown that the added water mass per foot of length for an ellipsoid is

$$M_w = CJ\pi B^2 \gamma_w \tag{2}$$

where C = Section inertia coefficient

B = Half beam or radius of ellipsoid at section

γ_w = Specific weight of fluid

$$J = \frac{\text{Actual K. E. surrounding fluid}}{\text{K. E. of fluid if flow is two dimensional}}$$

Now for a cylinder $C = 1$ and we can write

$$M_w = J\pi R^2 \gamma_w L \tag{3}$$

and

$$\frac{M_w}{M_c} = \frac{J\pi R^2 \gamma_w L}{\pi R^2 \gamma_w L} \tag{4}$$

so that

$$J = \frac{M_w}{M_c} \tag{5}$$

It was found that if Prof. Lewis' J factor is multiplied by our K, a fairly decent approximation to the actual frequency of a submerged body can be computed. A computation was made of the submerged frequency of a 42-inch body having 6-inch cones at each end of a cylindrical middle body 30 inches long.

Using Prof. Lewis' J factor multiplied by K for an L/D ratio of 21 at $\eta = .15$, the computed two noded frequency was 68.9 c.p.s. and the experimentally measured value was 70 c.p.s. For the three noded frequency the computed value was 187 c.p.s. and the measured value was 188 c.p.s.

Although this method gives very good results in the case tested, further investigation is required to determine its limits of application. Similar results would have been found using Dr. Schauer's M_w/M_c in a like manner.

$$M_w = \pi R^2 \gamma C J \tag{2}$$

where C = Section inertia coefficient

R = Half beam or radius of ellipse at section

γ_w = Specific weight of fluid

$J = \dfrac{\text{Actual K. E. surrounding fluid}}{\text{K. E. of fluid if flow is two dimensional}}$

Now for a cylinder $C = 1$ and we can write

$$M_w = J\pi R^2 \gamma_w L \tag{3}$$

and

$$\frac{J\pi R^2 \gamma_w L}{\pi R^2 \gamma_w L} = \frac{M_x}{M_e} \tag{4}$$

so that

$$\frac{M_x}{M_e} = J \tag{5}$$

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Although this method gives very good results in the case tested, further investigation is required to determine the limits of application. Similar results would have been found using Mr. Schauer's M_w/M_e in a like manner.

V. RECOMMENDATIONS

In view of the large discrepancy between values of theoretical mass found by analytical means and those measured, it is recommended that an investigation similar to this be made of families of bodies of revolutions having ends whose section decreases to zero.

The measurement of the virtual mass of bodies which do not have complete radial symmetry is much more difficult since pains must be taken to insure that the vibrations are limited to the plane desired. However, it is recommended that where data exists on the vibration of submerged bodies such as submarines, an attempt be made to apply the correction, K, to compute their frequencies.

RECOMMENDATIONS

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VI. APPENDIX

A. DETAILS OF PROCEDURE

Selection of Tubing

Since the object of this investigation is to determine the virtual added mass of water, it is desirable that the cylinder be made of a material having a low specific gravity. Thus the added water mass will be a large fraction of the total virtual mass of the body. In order that the frequencies of the cylinder will not be excessively high, the material should have a relatively low modulus of elasticity.

As a result of these considerations, it was decided that the cylinder should be constructed of Lucite. The specific gravity of Lucite is 1.18 and has a modulus of elasticity in the order of 5×10^5 pounds per square inch.

Effect of Added Masses

The weight of the soft iron wire was .373 ounces and covered three-fourths of an inch of the cylinder. The weight of the crystal pick-up and clamp used to hold it securely in place was .303 ounces. The weight of the plastic cylinder was .497 ounces per inch. The effect of these added masses on the frequencies was assumed negligible.

Comparison of Frequency Measurement

The high speed movie camera method of frequency measurement proved very satisfactory, but also required a great deal of time. The camera had a neon bulb timing light built in which was energized by a 1000 cycle audio oscillator. The output of the oscillator was amplified through a CRO which provided a means of adjusting the light to the desired brightness. The only errors

APPENDIX A

Selection of Lucite.

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As a result of these considerations, it was decided that the cylinder should be constructed of Lucite. The specific gravity of Lucite is 1.18 and has a modulus of elasticity in the order of 5×10^5 pounds per square inch.

Effect of Added Masses

The weight of the coil lead wire was .575 ounces and covered three-fourths of an inch of the cylinder. The weight of the crystal pick-up and clamp used to hold it securely in place was .903 ounces. The weight of the plastic cylinder was .497 ounces per inch. The effect of these added masses on the frequencies was assumed negligible.

Comparison of Frequency Measurement

The high speed movie camera method of frequency measurement proved very satisfactory, but also required a great deal of time. The camera had a neon bulb timing light built in which was energized by a 1000 cycle audio oscillator. The output of the oscillator was amplified through a CRO which provided a means of adjusting the light to the desired brightness. The only errors

that could be made in this method were (a) errors in counting the film and (b) frequency drift in the oscillator. Two separate runs were made on each length tested.

In order to speed up the experiment and reduce the amount of labor involved, it was decided to try frequency measurement with an electronic decade counter. If there is any frequency drift of the audio oscillator, the results of the decade counter will be more accurate than the movie camera since the decade counter records every second noting any change which may occur.

Boundary Effects in Water

To check for wall effects, the cylinder was submerged in the stability tank and the test rerun. The results obtained were the same as those observed in the towing tank test. A further check was made by varying the distance of the cylinder from the walls in the stability tank. Again no difference was noted. No attempt was made to vibrate the cylinder within four diameters of the wall.

Comparison with Different Cylinder

The necessity of re-checking the 30-inch cylinder arose after it had been cut down to 22 inches. A new 30-inch plastic tube was re-wound with approximately the same amount of wire as before. Both the air and water tests agreed identically with the previous 30-inch test.

Damping Effects of Rubber Band Stand-off

To ascertain whether the rubber band had any appreciable damping effect, the force necessary to stretch the rubber band one inch and the force required to deflect the tube one inch were calculated. To stretch the rubber band

[illegible] carried out [illegible] (1) [illegible] each length tested.

In order to speed up the experiment and reduce the amount of labor involved, it was decided to try frequency measurement with an electronic decade counter. If there is any frequency drift of the audio oscillator, the results of the decade counter will be zero counts when the movie camera since the decade counter records every second noting any change which may occur.

<u>Boundary Effects in Water</u>

To check for wall effects, the cylinder was submerged in the stability tank and the test repeated. The results obtained were the same as those observed in the towing tank test. A further check was made by varying the distance of the cylinder from the walls in the stability tank. Again no difference was noted. An attempt was made to vibrate the cylinder within four diameters of the wall.

<u>Comparison with Different Cylinder</u>

The necessity of re-testing the 30-inch cylinder arose after it had been cut down to 42 inches. A new 72-inch plastic tube was re-wound with approximately the same amount of wire as before. Both the air and water tests agreed identically with the previous 30-inch test.

<u>Damping Effects of Rubber [illegible]</u>

To ascertain whether the rubber [illegible] had any appreciable damping effect, the force necessary to stretch the rubber [illegible] one inch and the force required to [illegible] were [illegible]. To stretch the rubber [illegible]

one inch required a force of .61 pounds, and to deflect the cylinder one inch, a force of 454 pounds was necessary. The damping effect of the rubber band may be neglected.

Amplitude Measurement

The equipment used was not capable of measuring amplitudes. This is partly because the natural resonant period of the pick-up, 980 cycles per second, was within the range of the frequencies measured. The primary difficulty was that it was not possible to maintain a constant exciting force on the cylinder. There also was no means of accurately measuring the exciting force. A constant exciting force was not possible because a higher force was required to excite the higher modes, but this same force would cause the cylinder to be drawn hard against the faces of the electro-magnet poles at the lower modes. It was necessary therefore to start with relatively low driving forces at low modes and increase the force to excite the higher frequencies.

Computation of Virtual Mass

The virtual mass was computed assuming negligible damping so that

$$\frac{M_V}{M_T} = \left(\frac{f_1}{f_2} \right)^2 \tag{6}$$

where

M_V = Virtual Mass

M_T = Mass of Tube

f_1 = Frequency in Air

f_2 = Frequency in Water

hand was required.

Amplitude Measurement

The equipment used was not capable of measuring amplitudes. This is partly because the natural resonant period of the pickups, 500 cycles per second, was within the range of the frequencies measured. The primary difficulty was that it was not possible to maintain a constant exciting force on the cylinder. There also was no means of accurately measuring the exciting force. A constant exciting force was not possible because a higher force was required to excite the higher modes but this same force would cause the cylinder to be drawn hard against the faces of the electro-magnet poles at the lower modes. It was necessary therefore to start with relatively low driving forces at low modes and increase the force to excite the higher frequencies.

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The virtual mass was computed assuming negligible damping so that

$$\frac{M_v}{M_T} = \left(\frac{f_1}{f_2}\right)^2 \qquad (6)$$

where

M_v = Virtual Mass

M_T = Mass of Tube

f_1 = Frequency in Air

f_2 = Frequency in Water

Let

M_w = Added Water Mass

M_c = Displaced Water Mass

then

$$M_v = M_w + M_T \tag{7}$$

$$M_w = M_T \left[\left(\frac{f_1}{f_2} \right)^2 - 1 \right] \tag{8}$$

and

$$\frac{M_w}{M_c} = \frac{M_T}{M_c} \left[\left(\frac{f_1}{f_2} \right)^2 - 1 \right] \tag{9}$$

Thus the ratio ($\frac{M_w}{M_c}$) may be computed from the measured values of the frequencies.

Computation of K

Dr. H. M. Schauer[2] has derived an analytical expression for the mass ratio, $\frac{M_w}{M_c}$, as follows:

$$\frac{M_w}{M_c} = \frac{1}{1 + \eta \dfrac{iH_o(i\eta)}{-H_1(i\eta)}} \tag{10}$$

where H_o and H_1 are Hankel functions and other symbols as previously defined.

The variation of the measured mass ratio from the above will be a function of η and the $\frac{L}{D}$ ratio. This ratio can be expressed as a ratio, K

$$K = \frac{\text{measured mass ratio}}{\text{analytical mass ratio}}$$

$$= \frac{(M_w/M_c)_M}{(M_w/M_c)_A} \tag{11}$$

Let

M_v = virtual mass

M_o = displaced water mass

then

$$M_v = M_v + M_T \tag{7}$$

$$M_v = M_T\left[\left(\frac{f_1}{f_2}\right)^2 - 1\right] \tag{8}$$

and

$$\frac{M_v}{M_o} = \frac{M_T}{M_o}\left[\left(\frac{f_1}{f_2}\right)^2 - 1\right] \tag{9}$$

Thus the ratio $\left(\frac{M_v}{M_o}\right)$ may be computed from the measured values of the frequencies.

<u>Computation of K</u>

Dr. H. M. Schauer(2) has derived an analytical expression for the mass ratio, $\frac{M_v}{M_o}$, as follows:

$$\frac{M_v}{M_o} = \frac{1}{1 + \frac{\beta}{2}\,\frac{H_o(i\beta)}{-H_1(i\beta)}} \tag{10}$$

where H_o and H_1 are Hankel functions and other symbols as previously defined. The variation of the measured mass ratio from the above will be a function of β and the $\frac{l}{D}$ ratio. This ratio can be expressed as a ratio, K

$$K = \frac{\text{measured mass ratio}}{\text{analytical mass ratio}}$$

(11) [illegible] =

B. SUMMARY OF DATA AND CALCULATIONS

1. Definition of Symbols.

f_1 = Observed air frequency, cycles per second

f_2 = Observed water frequency, cycles per second

M_w = Added mass of water

M_e = Mass of displaced water

L = Length of cylinder, inches

D = Diameter of cylinder, inches

$\eta = \frac{D}{2L}(m + 1)$

m = Mode number

2. Calculated values of $\frac{M_w}{M_c}$.

MODE	f_1	f_2	$\frac{M_w}{M_c}$	η
1	61.7			
2	170.5	86.2	0.82	0.179
3	332.0	169.1	0.795	0.238
4	539.5	279.0	0.770	0.298
5	787.5	419.0	0.709	0.358

TABLE I

Test Results 52.7-inch Cylinder L/D = 26.35

MODE	f_1	f_2	$\frac{M_w}{M_c}$	η
1	104.5	54	.792	.157
2	290.5	151	.768	.236
3	562	295	.742	.314
4	893	476	.711	.392
5	1277	692	.682	.471

TABLE II

Test Results of 40-inch cylinder L/D = 20

MODE	f_1	f_2	$\frac{M_W}{M_c}$	η
1	114	59	0.773	0.166
2	313	163	0.761	0.249
3	602	319	0.727	0.332
4	943	509	0.685	0.415
5	1351	739.5	0.665	0.497

TABLE III

Test Results 38-inch Cylinder L/D = 19

MODE	f_1	f_2	$\frac{M_W}{M_c}$	η
1	183	97.5	0.720	0.209
2	496.5	268.5	0.689	0.314
3	936	512	0.665	0.419
4	1454	808	0.637	0.523

Table IV

Test Results 30-inch Cylinder L/D = 15

MODE	f_1	f_2	$\frac{M_M}{M_e}$	η
1	205	112	0.674	0.224
2	553	305	0.657	0.336
3	1036	581.5	0.622	0.448

TABLE V

Test Results 28-inch Cylinder L/D = 14

MODE	f_1	f_2	$\frac{M_M}{M_e}$	η
1	299	172	0.590	0.286
2	800	465	0.570	0.428
3	1457	855	0.556	0.571

TABLE VI

Test Results 22-inch Cylinder L/D = 11

η	$iH_o(i\eta)$	$-H_1(i\eta)$	$\frac{iH_o(i\eta)}{-H_1(i\eta)}$	$\eta\frac{iH_o(i\eta)}{-H_1(i\eta)}$	$1+\eta\frac{iH_o(i\eta)}{-H_1(i\eta)}$	$\frac{M_W}{M_e}$
.120	1.431	5.20	.275	.0330	1.0330	.968
.131	1.376	4.75	.290	.0380	1.0380	.963
.175	1.198	3.50	.342	.0599	1.0599	.943
.196	1.128	3.11	.363	.0711	1.0711	.934
.261	.9956	2.275	.438	.114	1.114	.898
.262	.9954	2.265	.439	.115	1.115	.897
.327	.8238	1.763	.437	.153	1.153	.867
.349	.7865	1.635	.481	.168	1.168	.856
.392	.7209	1.425	.506	.198	1.198	.835
.437	.6609	1.249	.529	.231	1.231	.812
.524	.5639	.9928	.568	.298	1.298	.770

TABLE VIII

Analytical Calculation of Virtual Mass

By Dr. H. M. Schauer[2]

$$\frac{M_W}{M_e} = \frac{1}{1+\eta\frac{iH_o(i\eta)}{-H_1(i\eta)}}$$

M_W = Added Water Mass

M_e = Mass of Displaced Water

M_T = Mass of Cylinder

η = $(m+1)\frac{a}{L}$

m = Mode Number

a = Radius

L = Length

η	$iH_o(i\eta)$	$-H_1(i\eta)$	$\frac{iH_o(i\eta)}{-H_1(i\eta)}$	$\eta\frac{iH_o(i\eta)}{-H_1(i\eta)}$	$1+\eta\frac{iH_o(i\eta)}{-H_1(i\eta)}$	$\frac{M_W}{M_e}$
.120	1.431	5.20	.275	.0330	1.0330	.968
.131	1.376	4.75	.290	.0380	1.0380	.963
.175	1.198	3.50	.342	.0599	1.0599	.943
.196	1.128	3.11	.363	.0711	1.0711	.934
.261	.9956	2.275	.438	.114	1.114	.898
.262	.9954	2.265	.439	.115	1.115	.897
.327	.8238	1.763	.437	.153	1.153	.867
.349	.7865	1.635	.481	.168	1.168	.856
.392	.7209	1.425	.506	.198	1.198	.835
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M_W = Added Water Mass

M_e = Mass of Displaced Water

M_T = Mass of Cylinder

$\eta = (m+1)\frac{a}{L}$

m = Mode Number

a = Radius

L = Length

β	[illegible]	(βl) [illegible]	[illegible]	[illegible]	[illegible]	$\frac{M_w}{M_o}$
.120	[illegible]	[illegible]	.275	.0330	1.0330	.968
.133	1.376	[illegible]	.290	.0380	1.0380	.963
.175	1.196	3.50	.342	.0599	1.0599	.943
.196	1.126	3.11	.355	.0711	1.0711	.934
.251	.9956	2.275	.438	.114	1.114	.898
.262	.9954	2.265	.439	.115	1.115	.897
.327	.8238	1.763	.467	.153	1.153	.867
.349	.7865	1.635	.481	.168	1.168	.856
.392	.7209	1.428	.506	.198	1.198	.835
.437	.6609	1.249	.529	.231	1.231	.812
.524	.5539	.9928	.558	.298	1.298	.770

TABLE VIII

Analytical Calculation of Virtual Mass

By Dr. [illegible] M. Scherer(2)

$$\frac{M_w}{M_o} = \frac{1}{1 + \frac{(\beta l)\, I_1(la)}{(\beta l)\, I_0(la)}}$$

M_w = Added Water Mass

M_o = Mass of Displaced Water

M_1 = Mass of Cylinder

$\beta = \frac{a}{l}(m+1)$

m = Mode number

a = Radius

l = Length

3. Calculated values of K.

η	$(M_w/M_c)_M$	$(M_w/M_c)_A$	K
.2	.811	.933	.870
.25	.792	.910	.870
.30	.772	.886	.870
.35	.750	.860	.870
.40	.730	.833	.876

TABLE IX (a)

Values of K L/D = 26.35

η	$(M_w/M_c)_M$	$(M_w/M_c)_A$	K
.2	.772	.933	.829
.25	.754	.910	.829
.30	.737	.886	.830
.35	.718	.810	.834
.40	.700	.833	.840
.45	.683	.805	.847
.50	.665	.776	.856

TABLE IX (b)

Values of K L/D = 19

η	$(M_w/M_e)_M$	$(M_w/M_e)_A$	K
.2	.72	.933	.772
.25	.707	.910	.776
.30	.694	.886	.784
.35	.682	.860	.793
.40	.668	.833	.802
.45	.656	.805	.816
.50	.643	.776	.829

TABLE IX (c)

Values of K L/D = 15

η	$(M_w/M_e)_M$	$(M_w/M_e)_A$	K
.2	.677	.933	.726
.25	.667	.910	.734
.30	.657	.886	.741
.35	.648	.860	.754
.40	.639	.833	.767
.45	.629	.805	.781
.50	.619	.776	.796

TABLE IX (d)

Values of K L/D = 14

β	$(M/M_c)_M$	$(M/M_c)_A$	F
.2	.72	.933	.772
.25	.707	.910	.776
.30	.694	.886	.784
.35	.682	.860	.793
.40	.668	.833	.802
.45	.656	.805	.816
.50	.643	.776	.829

TABLE IX (c)

Values of F L/D = 15

β	$(M/M_c)_M$	$(M/M_c)_A$	F
.2	.677	.933	.726
.25	.667	.910	.734
.30	.657	.886	.741
.35	.648	.860	.754
.40	.639	.833	.767
.45	.629	.805	.781
.50	.619	.776	.796

TABLE IX [illegible]

Values of F L/D = [illegible]

η	$(M_w/M_e)_M$	$(M_w/M_e)_A$	K
.2	.600	.933	.644
.25	.595	.910	.654
.30	.590	.886	.664
.35	.585	.860	.676
.40	.577	.833	.690
.45	.571	.805	.706
.50	.565	.776	.726

TABLE IX (e)

Values of K L/D = 11

η	$(M_w/M_e)_M$	$(M_w/M_e)_A$	K
.15	.800	.954	.839
.224	.782	.922	.849
.30	.753	.886	.849
.40	.712	.833	.855
.50	.674	.776	.869

TABLE IX (f)

Values of K L/D = 21

C. SAMPLE CALCULATIONS

1. Theoretical frequency of free-free bar vibrating in air.

$$f = \frac{m_n^2}{2\pi} \left[\frac{EK^2 g}{\gamma}\right]^{1/2}$$

where m_n = 4.730 for first mode

E = Modulus of elasticity

= 580,000 psi for Lucite (approx.)

γ = Specific gravity = 1.18

g = 386 in/sec^2

k = Radius of gyration of section

k^2 = .441

$$f = \frac{.0081}{6.28} \left[\frac{580,000 \times .441}{11.05 \times 10^{-5}}\right]^{1/2}$$

= 62.0 cycles per second

for 2 noded frequency of 52.7-inch cylinder.

2. Calculation of $\frac{M_w}{M_c}$.

$$\frac{M_w}{M_c} = \frac{M_T}{M_c}\left[\left(\frac{f_1}{f_2}\right)^2 - 1\right]$$

where M_w = Added water mass

M_c = Displaced water mass

M_T = Mass of tube

For second mode of 52.7-inch bar

$$W_T = \text{Wt/in.} \times L + \text{Wt. clamp} + \text{Wt. Pickup} + \text{Wt. Exciting wire}$$

$$M_T = \frac{.497 \times 52.7 + .373 + .303}{16 \times 32.2}$$

$$= 0.0521$$

$$M_e = \frac{\pi D^2 L \gamma_w}{4g}$$

$$= \frac{3.14 \times 4 \times 52.7 \times 62.4}{4 \times 32.2 \times 1728}$$

$$= 0.186$$

$$f_1 = 170.5$$

$$f_2 = 86.2$$

$$\frac{M_W}{M_e} = .280(2.92)$$

$$= 0.820$$

3. Calculation of K.

By definition

$$K = \frac{M_W/M_e \text{ measured}}{M_W/M_e \text{ analytically computed}}$$

For 52.7" cylinder

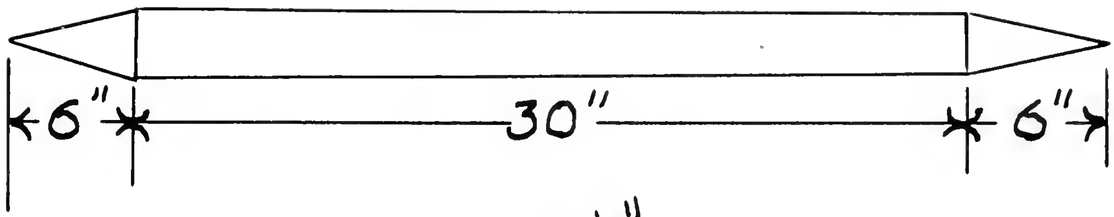
when $\eta = .2$

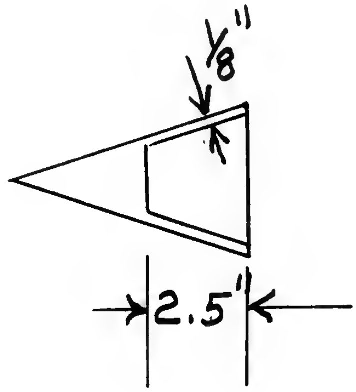
$$\left(\frac{M_W}{M_e}\right)_M = .811$$

$$\left(\frac{M_W}{M_e}\right)_A = .933$$

Hence $K = \frac{.811}{.933} = 0.870$

4.





The center of gravity of cones is approximately 3" from the base. Let us assume this body to be equivalent to a right circular cylinder 36 inches in length.

This assumption may be checked by calculating the air frequency of a 36-inch cylinder.

$$f = \frac{m_n^2}{2\pi} \left(\frac{EK^2 g}{\gamma} \right)^{1/2}$$

where $\frac{1}{2\pi} \left(\frac{EK^2 g}{\gamma} \right)^{1/2} = 7{,}350$

$m_n = \frac{4.730}{L}$ for 2 noded frequency

$f = \left(\frac{4.730}{36}\right)^2 \; 7350$

$f = 127$ cps

Observed frequency was 131 cps, therefore equivalent length is 35.5 inches.

The w ter frequency now may be computed using Prof. Lewis' J, corrected by K for a 42-inch right circular cylinder

$$f = \left(\frac{4.730}{35.5} \right)^2 (7350) \left(\frac{M_T}{M_T+M_W} \right)^{1/2}$$

where $M_T = .036$

$M_W = JKM_c$ (12)

$= (.947)(.839)(.120)$

$M_W = .0952$

$M_W + M_T = .1312$

$f = 68.9$ cps.

The experimental results gave a water frequency of 70 cps.

For the second mode

$J = .923$

$K = .849$

$M_W = .0940$

$M_W + M_T = .1300$

$f = 187$ cps

Experimental results gave a water frequency of 188 cps.

[illegible] corrected

by [illegible]

$$[illegible] \left(\frac{M_I}{M_w + M_I} \right)^{1/2}$$

where M_I = .190

A_w = .478c (12)

= (.947) (.839) (1.20)

M_w = .0952

$M_I + M_w$ = 1.12

f = 65.9 cps.

The experimental results gave a water frequency of 75 c.p.s.

For the second mode

I = .925

K = .849

M_w = .0940

$M_w + M_I$ = 1.1300

f = 147 cps

Experimental results gave a water frequency of 152 c.p.s.

D. ORIGINAL DATA

1. The following data was taken using the decade counter.

MODE	FREQUENCIES			AVERAGE
1	114	114	114	114
2	313	313	313	313
3	604	601	602	602
4	944	942	942	943
5	1352	1350	1351	1351

TABLE X

Air Frequencies 38-inch Cylinder

- -

MODE	FREQUENCIES			AVERAGE
1	59	59	59	59
2	163	163	163	163
3	320	319	319	319
4	510	507	509	509
5	740	739	739.5	739.5

TABLE XI

Water Frequencies 38-inch Cylinder

MODE	FREQUENCIES				AVERAGE
1	182	183	183	183	183
2	496.5	495.5	496.5	496.5	496.5
3	933	936	935.5	936.5	936.0
4	1454.5	1453.5	1453	1454.5	1454.0

TABLE XII

Air Frequencies 30-inch Cylinder

- -

MODE	FREQUENCIES				AVERAGE
1	98	97.5	97.5	97.5	97.5
2	268.5	269	267	268.5	268.5
3	510	513	513.5	511	512.0
4	810	809	808	807.5	808

TABLE XIII

Water Frequencies 30-inch Cylinder

MODE	FREQUENCIES		AVERAGE
1	205	205	205
2	553	552.5	553
3	1036	1035.5	1036

TABLE XIV

Air Frequencies 28-inch Cylinder

MODE	FREQUENCIES		AVERAGE
1	112	112	112
2	305	305	305
3	581.5	581.5	581.5
4	916	917	916.5

TABLE XV

Water Frequencies 28-inch Cylinder

MODE	FREQUENCIES			AVERAGE
1	298	299.5	298.5	299
2	800	801	800	800
3	1457	1458	1457	1457

TABLE XVI

Air Frequencies 22-inch Cylinder

- -

MODE	FREQUENCIES			AVERAGE
1	171	171	172	171
2	465	466	465	465
3	854	855	855	855

TABLE XVII

Water Frequencies 22-inch Cylinder

MODE	FREQUENCIES				AVERAGE
1	104	105	105	104	104.5
2	291	290	291	290	290.5
3	561.5	562	562	562	562
4	891	894	893	893	893
5	1278	1276	1276	1277	1277

TABLE XVIII

Air Frequencies 40-inch Cylinder

MODE	FREQUENCIES				AVERAGE
1	54.0	54.0	53.5	54.0	54.0
2	150.0	151.0	151	151	151
3	296	294	295	295	295
4	477	476.5	476	476	476
5	693	691	692	691.5	692

TABLE XIX

Water Frequencies 40-inch Cylinder

MODE	FREQUENCIES			AVERAGE
1	131	131	131	131
2	353	351	352	352
3	659	658	658	658
4	1016	1020	1014	1017
5	1417	1419	1415	1417

TABLE XX

Air Frequencies 30-inch Cylinder

with 6-inch Conical Ends

- -

MODE	FREQUENCIES			AVERAGE
1	70	70	70	70
2	188	188	188	188
3	358	355	352	355
4	555	555	555	555
5	796	790	790	792

TABLE XXI

Water Frequencies 30-inch Cylinder

with 6-inch Conical Ends

2. The following data was taken using the high speed movie camera.

MODE	FREQUENCIES		AVERAGE
1	61.7	61.7	61.7
2	170.5	170.5	170.5
3	332.0	332.0	332.0
4	539.5	539.5	539.5
5	787.5	787.5	787.5

TABLE XXII

Air Frequencies 52.7-inch Cylinder

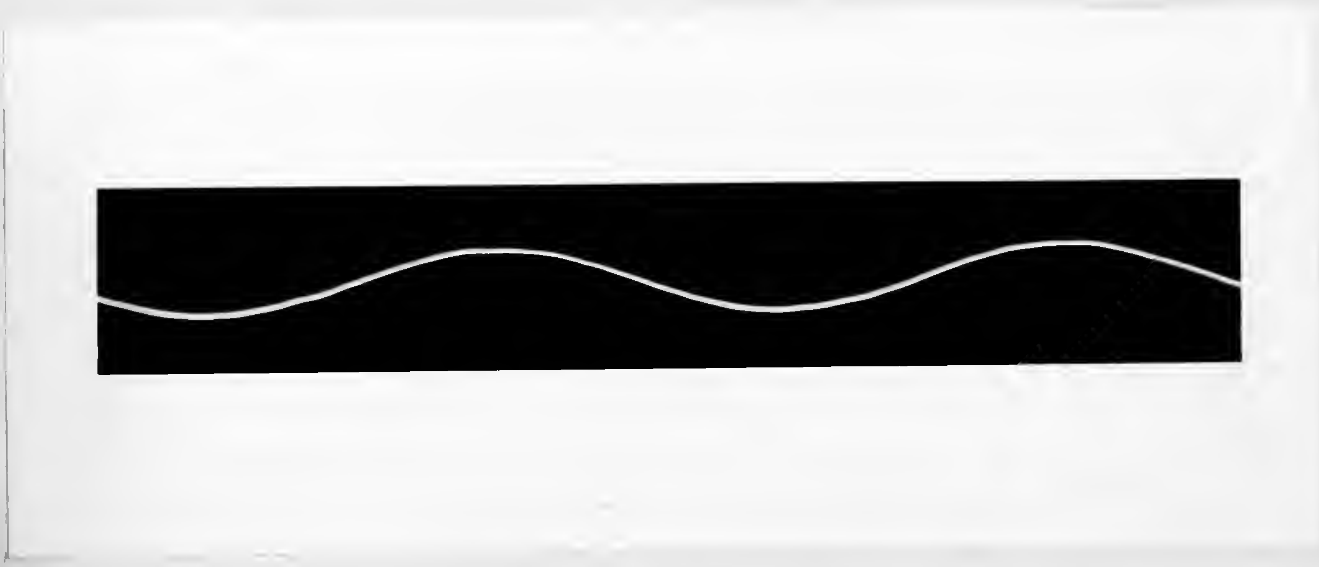
MODE	FREQUENCIES		AVERAGE
1			
2	86.2	86.2	86.2
3	169.1	169.1	169.1
4	279.0	279.0	279.0
5	419.0	419.0	419.0

TABLE XXIII

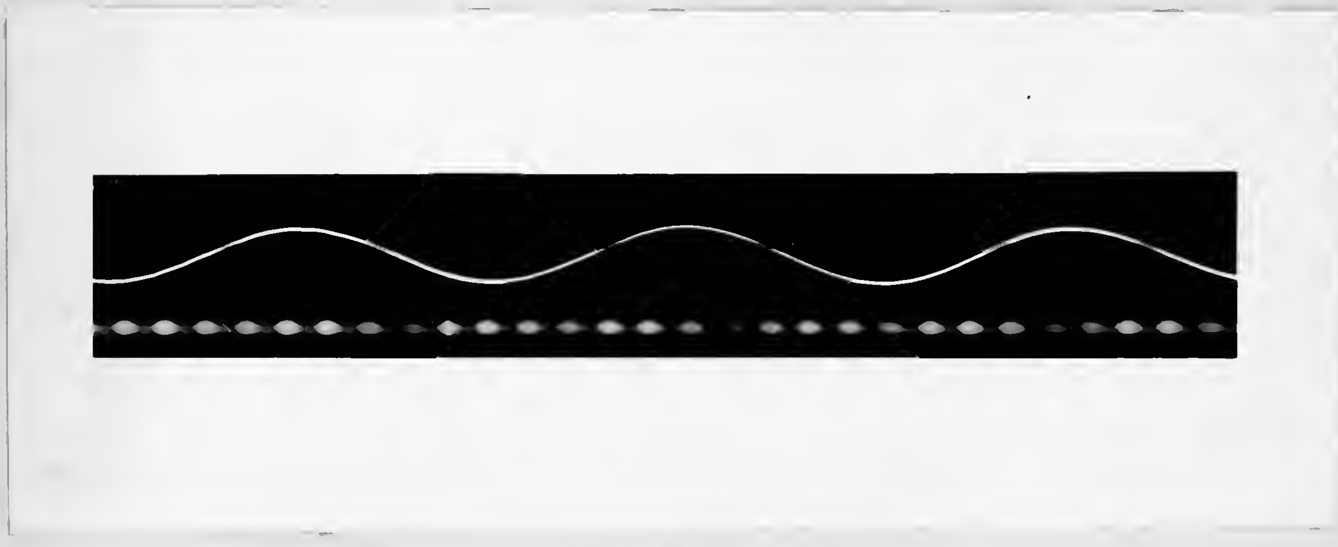
Water Frequencies 52.7-inch Cylinder

FIGURE VI

PHOTOGRAPHS OF TYPICAL TWO NODED FREQUENCIES



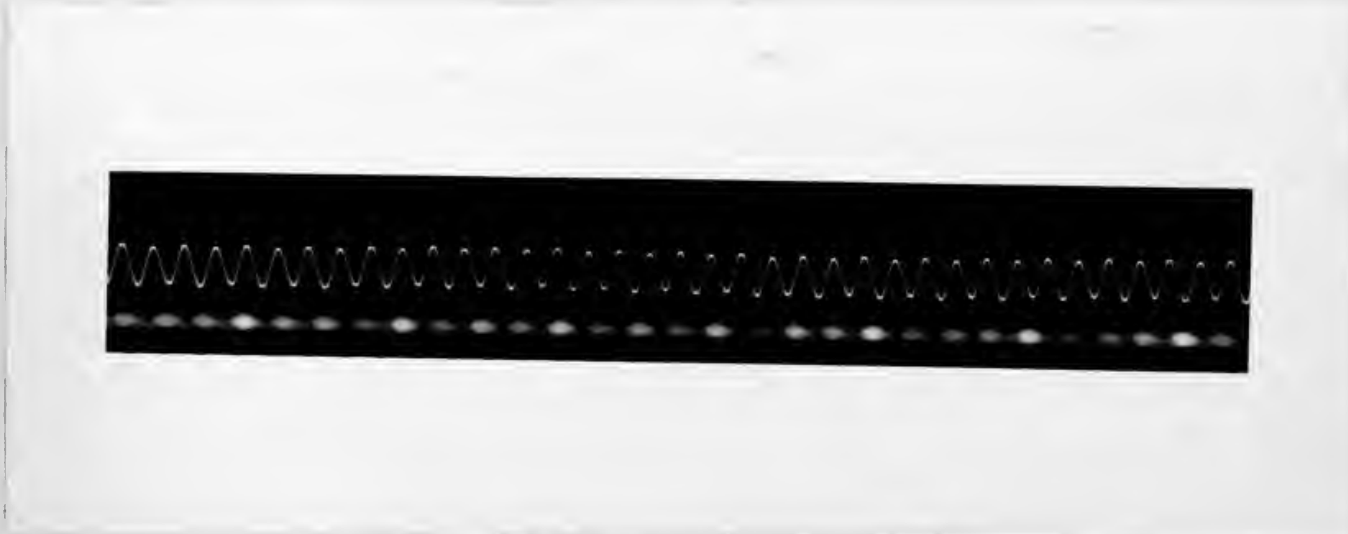
TWO NODED WATER FREQUENCY



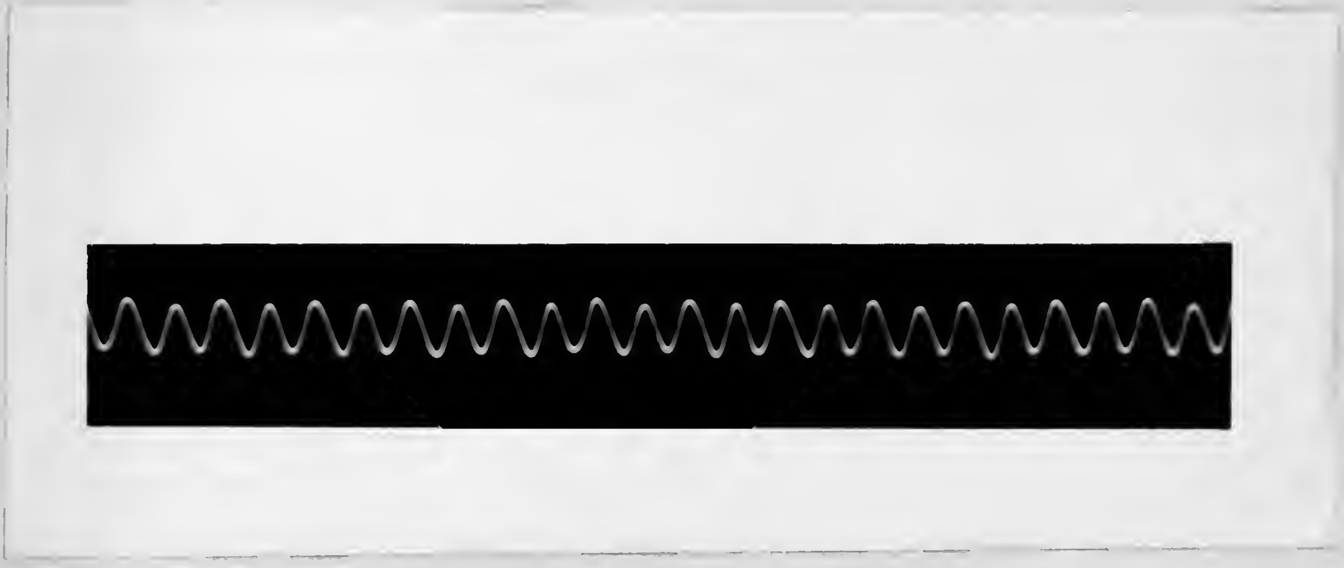
TWO NODED AIR FREQUENCY

FIGURE VII

PHOTOGRAPHS OF TYPICAL SIX NODED FREQUENCIES



SIX NODED AIR FREQUENCY



SIX NODED WATER FREQUENCY

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Thesis
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Rogers
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20519